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# Coast Guard drug interdiction: a renewal-reward approach to determine optimum investigation time

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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS

C75415

COAST GUARD DRUG INTERDICTION: A  
RENEWAL-REWARD APPROACH TO DETERMINE  
OPTIMUM INVESTIGATION TIME

by

Eric A. Copeland

March 1988

Thesis Advisor:

Donald P. Gaver

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Coast Guard Drug Interdiction: A  
Renewal-Reward Approach To Determine  
Optimum Investigation Time

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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## ABSTRACT

A renewal-reward model is developed to predict the optimum amount of time that Coast Guard personnel should spend investigating a vessel for illicit substances. The optimal investigation time is determined with respect to three criteria; maximizing the number of arrests, maximizing the quantity of drugs confiscated, and minimizing the quantity of drugs that escape detection. A simulation study indicates that the optimal investigation time is very sensitive to underlying distributional assumptions. The basic service system model may have wider application, i.e., to combat modelling, where it may be desirable to investigate a potential target to estimate its value before committing limited resources. An adaption of the model may also be of help in allocating resources for mineral exploration.

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## I. INTRODUCTION

One duty of the United States Coast Guard is the interdiction of drug smugglers. When Coast Guard personnel board a suspect vessel to search for contraband, a decision must be made regarding the amount of time allocated for the investigation. If a careful, but lengthy, search is conducted, there is a high probability of finding contraband if it is present. However, a lengthy investigation incurs a penalty. While Coast Guard personnel are searching one vessel, other shipping, which could have contraband onboard, is passing through the patrol area without being examined. At the other extreme, if the suspect vessel is examined in a cursory manner, the Coast Guard Vessel (CGV) will be able to stop and search more shipping but there is a higher probability of not discovering the contraband, if present, because of the short investigation time. The purpose of this thesis is to develop a renewal-reward model to determine the optimum investigation time.

## II. RENEWAL-REWARD MODEL

Consider the following scenario. All shipping encountered by a patrolling CGV is stopped and searched to determine if contraband is onboard. After leaving port, a CGV searcher a time  $S_1$  until a vessel is sighted. A detected vessel has a probability  $P_b$  of being Bad (having contraband onboard) and  $P_g = 1 - P_b$  of being Good (having no contraband onboard). A Bad vessel contains the random number  $J = j(1,2,3,\dots)$  units of illicit substance with probability  $b_j$ . The time required to find the first unit of the contraband, given that  $J$  units are present, is modelled here as the minimum of  $J$  independent identically distributed times having distribution function  $F_2(t)$ ; this model is illustrative only, and may be altered in various realistic directions. If time  $T$  is required to locate an incriminating unit, then

$$P\{T > t | J=j\} = [1-F_2(t)]^j ; j=1,2,3,\dots$$

so, upon removing the condition,

$$1-F_T(t) = P\{T > t\} = \sum_{j=1}^{\infty} [1-F_2(t)]^j b_j ; j=1,2,3,\dots \quad (2.1)$$

Adopt the following decision rule: Establish a predetermined investigation time  $L$ . When a vessel is detected, it is stopped and searched. If no contraband is

discovered in the investigation time  $L$ , the vessel is released and the CGV resumes patrol. If any contraband is discovered before the end of the investigation time  $L$ , the vessel is detained and escorted to port for further search and investigation. A time period  $D$  is required to escort the vessel to base. The patrol cycle ends when the escorted vessel arrives at the base.

The long run average reward per cycle,  $R$ , can be calculated in the following manner [Ross: pp. 279-294].

$$R = \frac{E[R_c]}{E[C]} \quad (2.2)$$

where:  $E[R_c]$  is the expected reward for a patrol cycle.

$E[C]$  is the expected duration of a patrol cycle.

The reward associated with an apprehension can be defined in various ways. Three different reward criteria will be examined. For each of the three cases, we will determine the optimal investigation time  $L$  which maximizes the long run average cycle reward.

#### A. EXPECTED DURATION OF PATROL CYCLE: $E[C]$

We must distinguish between the following two cases for the investigation time  $I$ :

$I_a$  - an investigation time that results in the release of the vessel.

$I_b$  - an investigation time that results in the detention of the vessel.

Where

$$I = \begin{cases} I_g = L & \text{with probability } P_g + P_b \overline{F_T}(L) = \alpha \\ I_b = T & \text{with probability } P_b F_T(L) = \overline{\alpha} \end{cases}$$

The length of a CGV patrol cycle can be represented as:

$$C = \begin{cases} S_1 + I_b + D & \text{With probability } \overline{\alpha}(L) \\ S_1 + I_g + C' & \text{With probability } \alpha(L) \end{cases}$$

where  $C'$  has the same unconditional distribution as  $C$ , for if the first investigation results in a release of the vessel being searched, the process re-starts (regenerates).

The expected length of a patrol cycle can be expressed as:

$$E[C] = E[S] + (E[I_b] + E[D])\overline{\alpha} + (L + E[C'])\alpha \quad (2.3)$$

or

$$E[C] = \frac{E[S] + E[D]\overline{\alpha} + E[I_b]\overline{\alpha} + L\alpha}{1 - \alpha} \quad (2.4)$$

which can be written as

$$E[C] = E[D] + E[I_b] + \frac{E[S]}{\alpha} + L\frac{\alpha}{\alpha} \quad (2.5)$$

Solving for the term  $E[I_b]$  yields

$$E[I_b] = \frac{\int_0^L t F_T(dt)}{F_T(L)} \quad (2.6)$$

Integration by parts shows that

$$\int_0^L \overline{F_T}(t) dt = \overline{F_T}(L)L + \int_0^L t F_T(dt) \quad (2.7)$$

Hence

$$E[I_b] = \frac{\int_0^L \overline{F_T}(t) dt - L\overline{F_T}(L)}{F_T(L)} \quad (2.8)$$

Therefore

$$E[C] = \frac{E[S] + E[D]P_b F_T(L) + P_b \int_0^L \overline{F_T}(t) dt + LP_g}{P_b F_T(L)} \quad (2.9)$$

which reduces to

$$E[C] = E[D] + \frac{E[S]}{P_b F_T(L)} + \frac{\int_0^L \overline{F_T}(t) dt}{F_T(L)} + \frac{P_g L}{P_b F_T(L)} \quad (2.10)$$

#### B. REWARD CRITERIA 1

Suppose the CGV is rewarded for making an arrest.

Since only one arrest is made per cycle,

$$E[R_c] = 1 \quad (2.11)$$

and from equation 2.2

$$R = \frac{1}{E[C]} \quad (2.12)$$



Consider the following example

$$\begin{aligned}
 F_2(t) &= 1 - e^{-\mu t} & , \mu &= 0.08 \\
 S(t) &= 1 - e^{-\lambda_1 t} & , \lambda_1 &= 1.1 \\
 D(t) &= 1 - e^{-\lambda_2 t} & , \lambda_2 &= 0.3 \\
 b_j &= (1 - \beta)\beta^{j-1} & , \beta &= 0.8; j = 1, 2, \dots \\
 P_b &= 0.2
 \end{aligned}$$

From equation 2.1

$$\overline{F_T}(t) = \sum_{j=1}^{\infty} (e^{-\mu t})^j (1-\beta)\beta^{j-1} = \frac{e^{-\mu t}(1-\beta)}{1 - \beta e^{-\mu t}} \quad (2.13)$$

and

$$F_T(t) = 1 - \overline{F_T}(t) = \frac{1 - e^{-\mu t}}{1 - \beta e^{-\mu t}} \quad (2.14)$$

From equation 2.10, the expected duration of the patrol cycle is

$$\begin{aligned}
 E[C] &= \frac{1}{\lambda_2} + \frac{1/\lambda_1}{P_b \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} + \frac{\left( \frac{1-\beta}{\beta \mu} \right) \text{Ln} \left( \frac{1-\beta e^{-\mu L}}{1-\beta} \right)}{\left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} \\
 &\quad + \frac{P_g L}{P_b \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} \quad (2.15)
 \end{aligned}$$

and from equation 2.12, the long run average reward per cycle can be calculated.

$$R = \frac{\left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right) \lambda_1 \lambda_2 P_b}{P_b \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right) \lambda_1 + \lambda_2 + \lambda_1 \lambda_2 P_b \left( \frac{1-\beta}{\beta \mu} \right) \text{Ln} \left( \frac{1-\beta e^{-\mu L}}{1-\beta} \right) + P_g L \lambda_1 \lambda_2} \quad (2.16)$$

### C. REWARD CRITERIA 2

The reward assigned to the CGV is the amount of drugs confiscated.

$$E[R_c] = E[J | T \leq L] \quad (2.17)$$

The probability that a vessel being searched contains  $J = j$  units of drugs and that the drugs are discovered is

$$P\{(T \leq L) \wedge (J=j)\} = \left[ 1 - (1-F_2(L))^j \right] b_j \quad (2.18)$$

so

$$P\{T \leq L\} = F_T(L) = \sum_{j=1}^{\infty} \left[ 1 - (1-F_2(L))^j \right] b_j \quad (2.19)$$

and thus

$$P\{(J=j) | (T \leq L)\} = \frac{\left[ 1 - (1-F_2(L))^j \right] b_j}{F_T(L)} \quad (2.20)$$

and

$$E[J | T \leq L] = \frac{\sum_{j=1}^{\infty} j \left[ 1 - (1-F_2(L))^j \right] b_j}{F_T(L)} \quad (2.21)$$

using the parameters from the Example in Model 1,

$$E[J|F \leq L] = \frac{\sum_{j=1}^{\infty} j \left[ 1 - (e^{-\mu L})^j \right] (1-\beta) \beta^{j-1}}{(1-e^{-\mu L}) / (1-\beta e^{-\mu L})} \quad (2.22)$$

which can be evaluated as in the previous example as

$$E[J|T \leq L] = \frac{(1-\beta) \left[ \left( \frac{1}{(1-\beta)^2} \right) - \left( \frac{e^{-\mu L}}{(1-\beta e^{-\mu L})^2} \right) \right]}{(1-\beta) \left[ \left( \frac{1}{(1-\beta)} \right) - \left( \frac{e^{-\mu L}}{(1-\beta e^{-\mu L})} \right) \right]} \quad (2.23)$$

using equation 2.2, the long run average reward is

$$R = \frac{E[J|T \leq L]}{E[C]}$$

or

(2.24)

$$R = \frac{\left[ \left( \frac{1}{(1-\beta)^2} \right) - \left( \frac{e^{-\mu L}}{(1-\beta e^{-\mu L})^2} \right) \right]}{\left[ \left( \frac{1}{(1-\beta)} \right) - \left( \frac{e^{-\mu L}}{(1-\beta e^{-\mu L})} \right) \right]}$$

$$\frac{1}{\lambda_2} + \frac{\frac{1}{\lambda_1}}{P_b \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} + \frac{\left( \frac{1-\beta}{\beta \mu} \right) \text{Ln} \left( \frac{1-\beta e^{-\mu L}}{1-\beta} \right)}{\left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} + \frac{P_c L}{P_b \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)}$$

#### D. REWARD CRITERIA 3

The reward examined in model three is the difference between the quantity of drugs confiscated and the quantity

of drugs that escape detection. The expected reward in a cycle is

$$R_c = E[J|T \leq L] - E[M]$$

where M is defined as the quantity of drugs that escape detection in a cycle.

$$M = \begin{cases} J_{IB} + J_D & \text{With probability } P_b F_T(L) \\ J_{IG} + J_0 + M & \text{With probability } P_g + P_b \overline{F_T(L)} \end{cases}$$

where:

$J_{IB}$  = Drug quantity passing through patrol area while CGV searches a Bad ship

$J_D$  = Drug quantity passing through patrol area while CGV escorts a Bad ship to shore

$J_{IG}$  = Drug quantity passing through patrol area while CGV searches a Good ship

$J_0$  = Drug quantity onboard a ship classified as Good that is actually Bad

If conditional expectations are taken assuming that the Bad vessels pass through the region according to a Poisson process with rate  $\lambda_1 P_b$ , then

$$E[M] = \lambda_1 P_b E[J] (E[I_b] + E[D]) P_b F_T(L) + (\lambda_1 P_b E[J] E[I_g] + E[M]) (P_g + P_b \overline{F_T(L)}) + E[J_0] P_b \overline{F_T(L)} \quad (2.25)$$

or

$$E[M] = \frac{\lambda_1 P_b E[J] \left\{ P_b \left( \int_0^L \overline{F_T}(L) dt - L \overline{F_L} \right) + E[D] P_b F_T(L) + L \left( P_g + P_b \overline{F_T}(L) \right) \right\}}{P_b F_T(L)} + \left( \frac{E[J_0] P_b \overline{F_T}(L)}{P_b F_T(L)} \right) \quad (2.26)$$

where  $E[J_0]$  is defined as

$$E[J|T>L] = \frac{\sum_{j=1}^{\infty} j (1-F_2(L))^j b_j}{\overline{F_T}(L)}$$

Using the distributions defined in the Example for Model 1

$$E[M] = \frac{P_b \lambda_1}{(1-\beta) \left( \frac{1-e^{-\mu L}}{1-\beta e^{-\mu L}} \right)} \left[ \frac{1-\beta}{\beta \mu} \ln \left( \frac{1-\beta e^{-\mu L}}{1-\beta} \right) + \frac{1-e^{-\mu L}}{\lambda_2 (1-\beta e^{-\mu L})} + \frac{L P_g}{P_b} \right] + \frac{(1-\beta) e^{-\mu L}}{(1-\beta e^{-\mu L}) (1-e^{-\mu L})} \quad (2.27)$$

and the long run average reward can be expressed as

$$R = \frac{\text{Equation [2.21]} - \text{Equation [2.27]}}{\text{Equation [2.15]}} \quad (2.28)$$



### III. RESULTS

The optimum investigation time was determined for the above example for each reward criteria using two methods. The first method consisted of writing a Monte Carlo simulation of the patrol cycle using the distributions of the example and running the simulation using various values of the investigation time  $L$ . Statistics were gathered during the simulation allowing the calculation of the long run average reward  $R$ . The investigation time was varied from zero to six hours in increments of two tenths of an hour. The optimum investigation time was determined by graphing the long run average reward as a function of investigation time and finding the value of  $L$  which maximized  $R$ . Each cycle was replicated 20,000 times. A detailed discussion of the simulation can be found in Appendix A.

The second method consisted of writing a computer program for the three equations representing the long run average reward for the three reward criteria for the example and solving the equations for various values of the investigation time  $L$ . As in the case of the simulation, the value of  $L$  was varied from zero to six hours in increments of two tenths of an hour. The optimum investigation time was again determined by graphing the

long run average reward as a function of investigation time and finding the value of  $L$  which maximized  $R$ .

Figures 1,2, and 3 contain the results of both the simulation and numerical solution using the distributions of the examples for reward criteria 1,2, and 3 respectively. The solid line represents the analytical solution and the circles are the simulation results. The maximum long run average reward using criteria 1 can be achieved by using an investigation time between 1.4 and 2.2 hours. The maximum long run average reward using criteria 2 and 3 can be achieved using an investigation time between 1.2 and 1.4 hours. It is interesting to note that, using the distributions presented in these examples, and investigation time exists that maximizes all three reward criteria simultaneously. This occurs at 1.4 hours for the input distributions. We do not know that this state of affairs will persist.

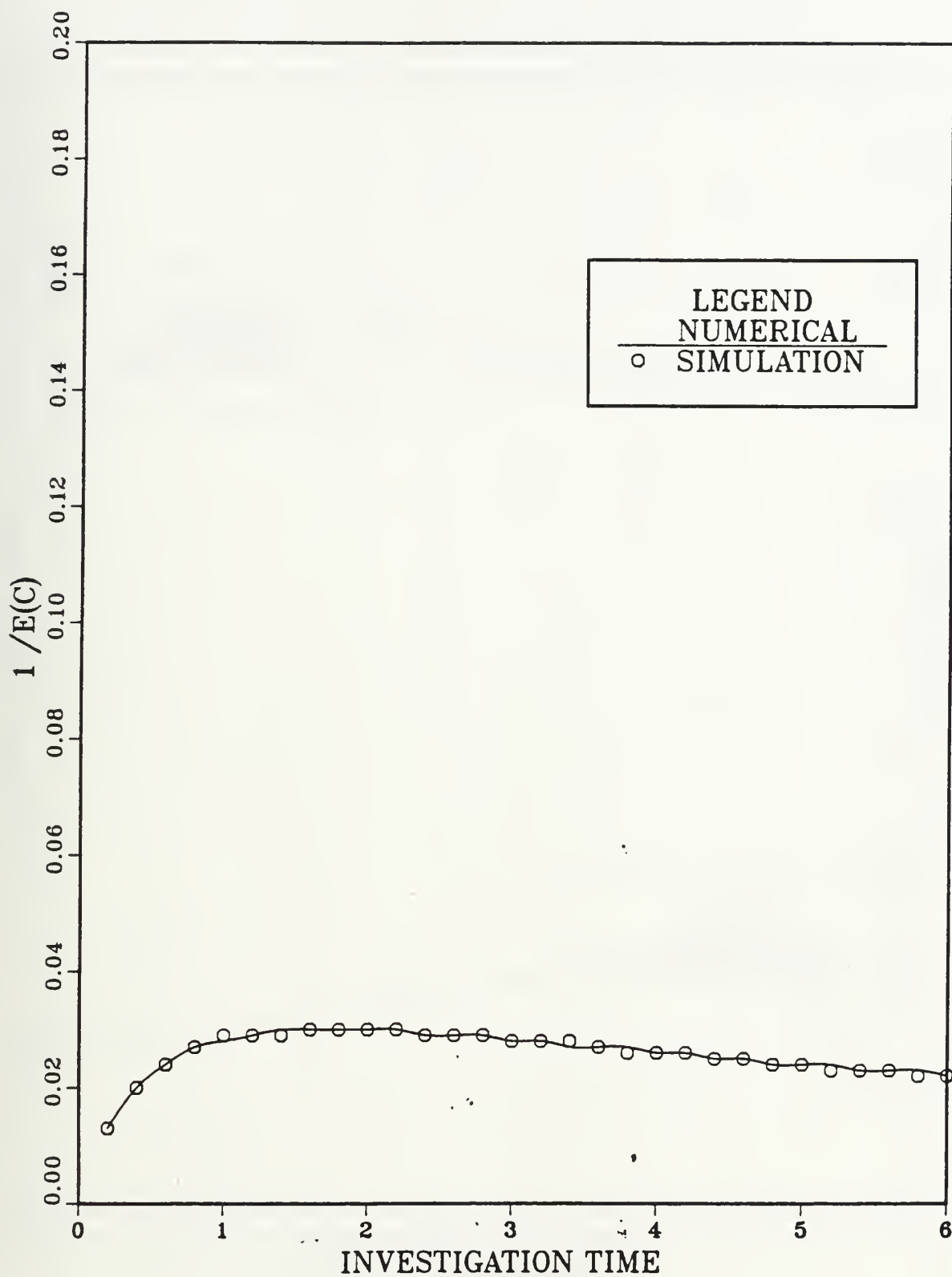


Figure 1: Reward-Criteria 1 Results

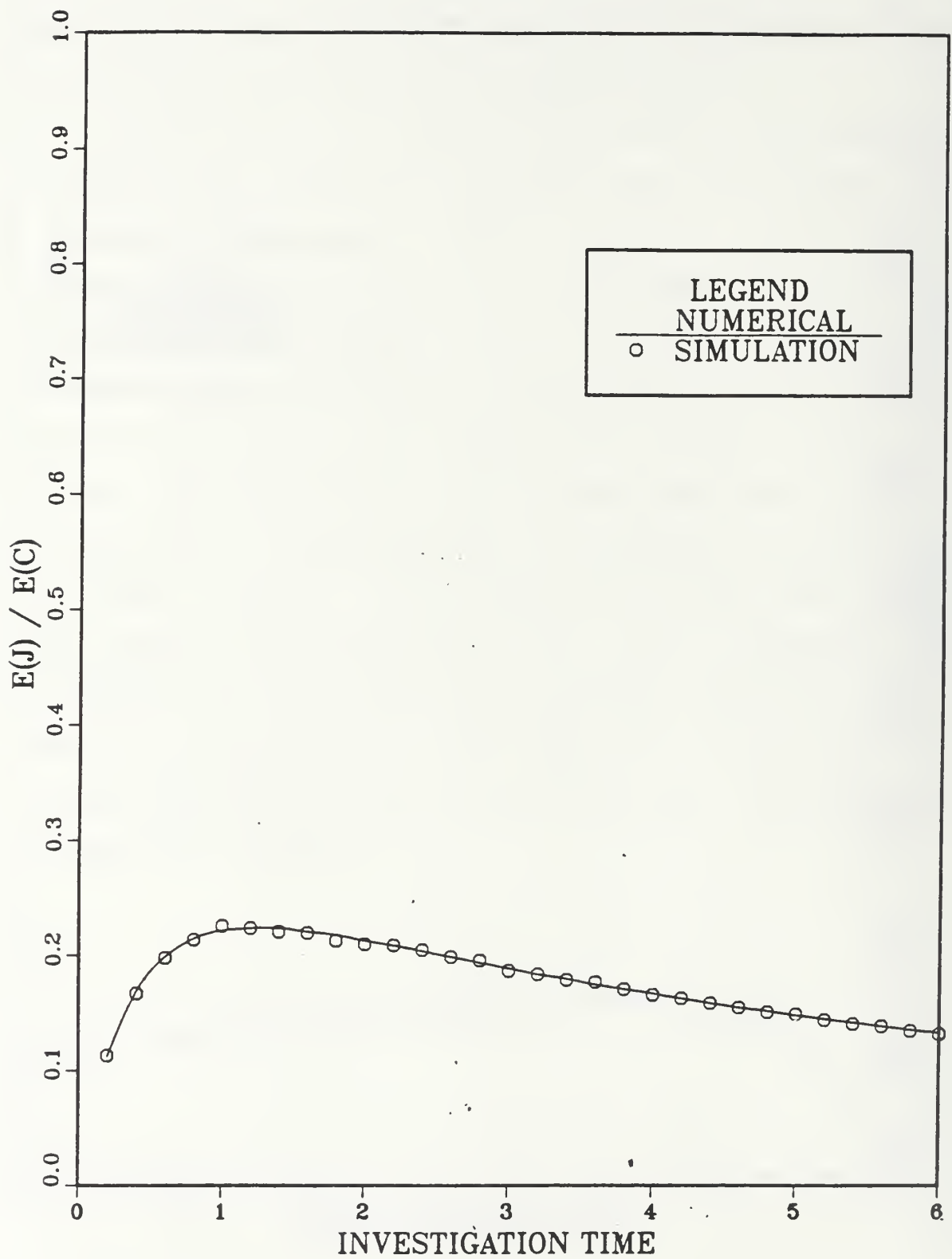


Figure 2: Reward Criteria 2 Results

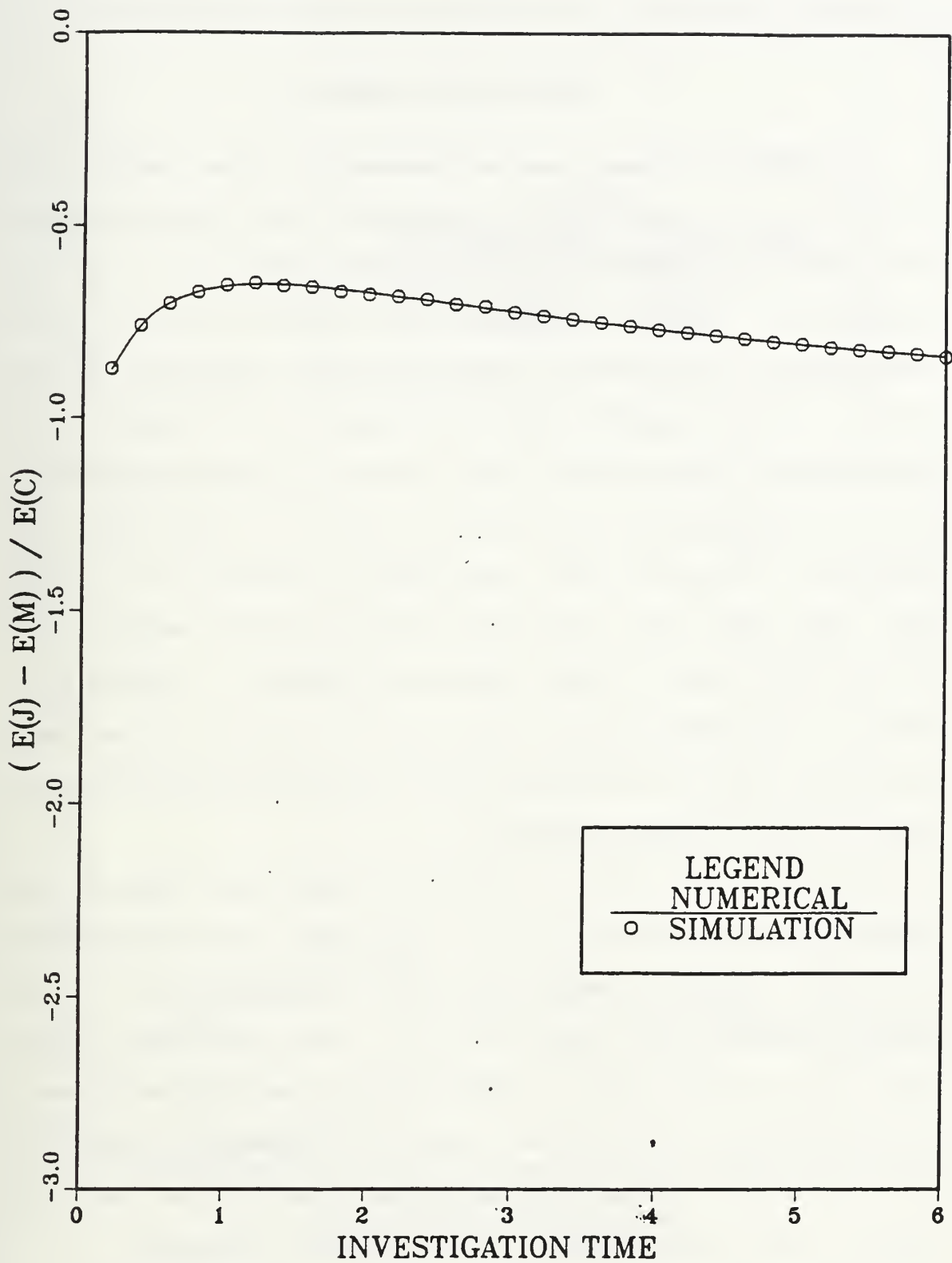


Figure 3: Reward Criteria 3 Results



#### IV. SENSITIVITY ANALYSIS

In order to check the robustness of the model with respect to the underlying assumptions, the simulation was run exactly as before with the exception that  $F_2(t)$  is now assumed to have a lognormal distribution instead of an exponential distribution. Each cycle was replicated 20,000 times. Three separate cases are examined. In case A, the distribution function  $F_2(t)$  has the same mean and variance as the exponential distribution used in the previous examples. In case B, the distribution function  $F_2(t)$  has the same mean but twice the variance as the exponential distribution used in the previous examples. Finally, in case C, the distribution function  $F_2(t)$  has the same mean but four times the variance as the exponential distribution used in the previous examples.

Figures 4,5, and 6 contain a comparison between the exponential base case and the lognormal cases mentioned above for reward criteria 1,2, and 3 respectively. It is readily apparent that the results for the exponential base case and lognormal case A, with the same mean and variance, are different. Furthermore, within the lognormal family of curves, it can be seen by examining case B and C that the results vary significantly as the variance increases.

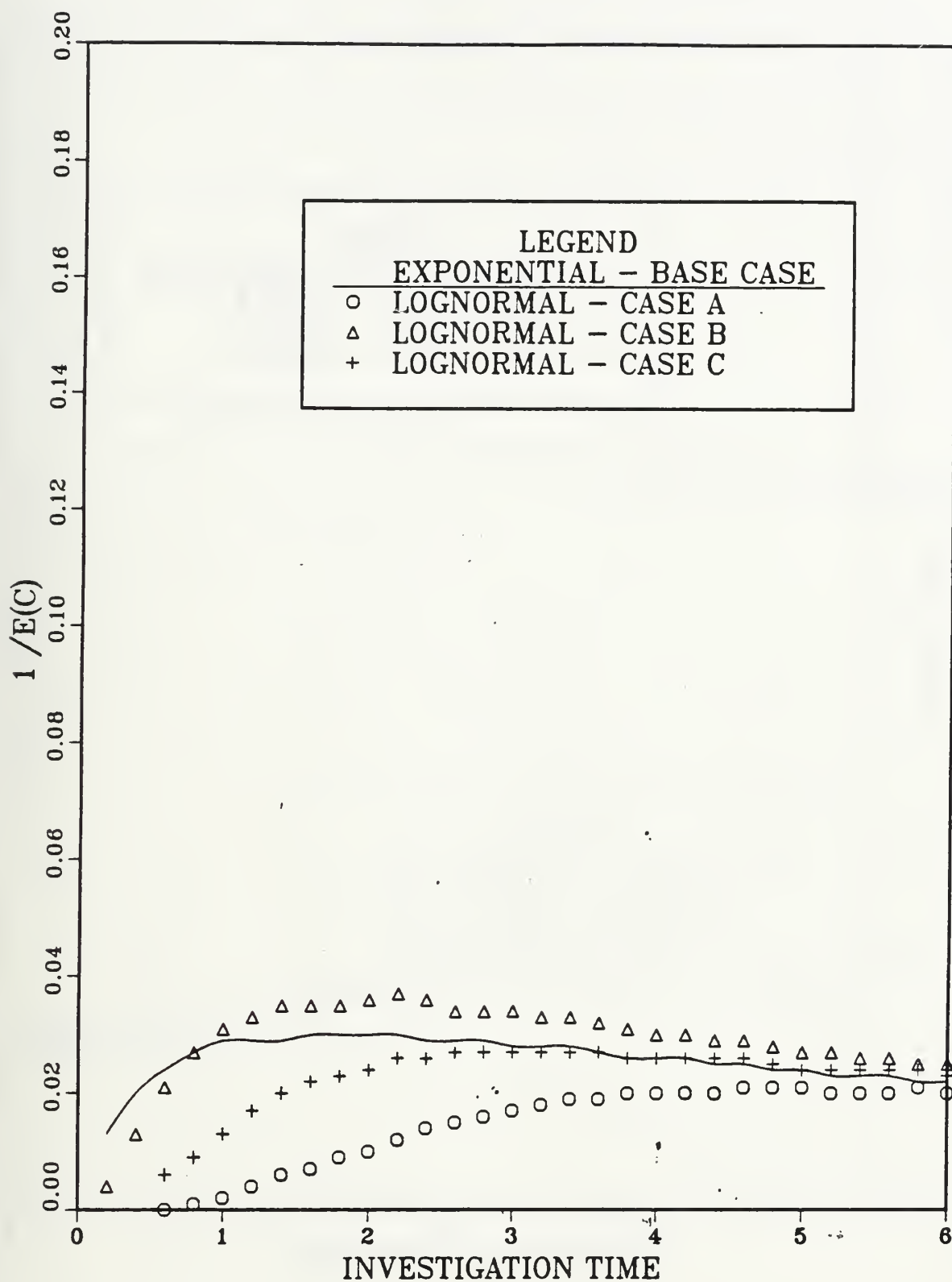


Figure 4: Reward Criteria 1 Sensitivity Analysis

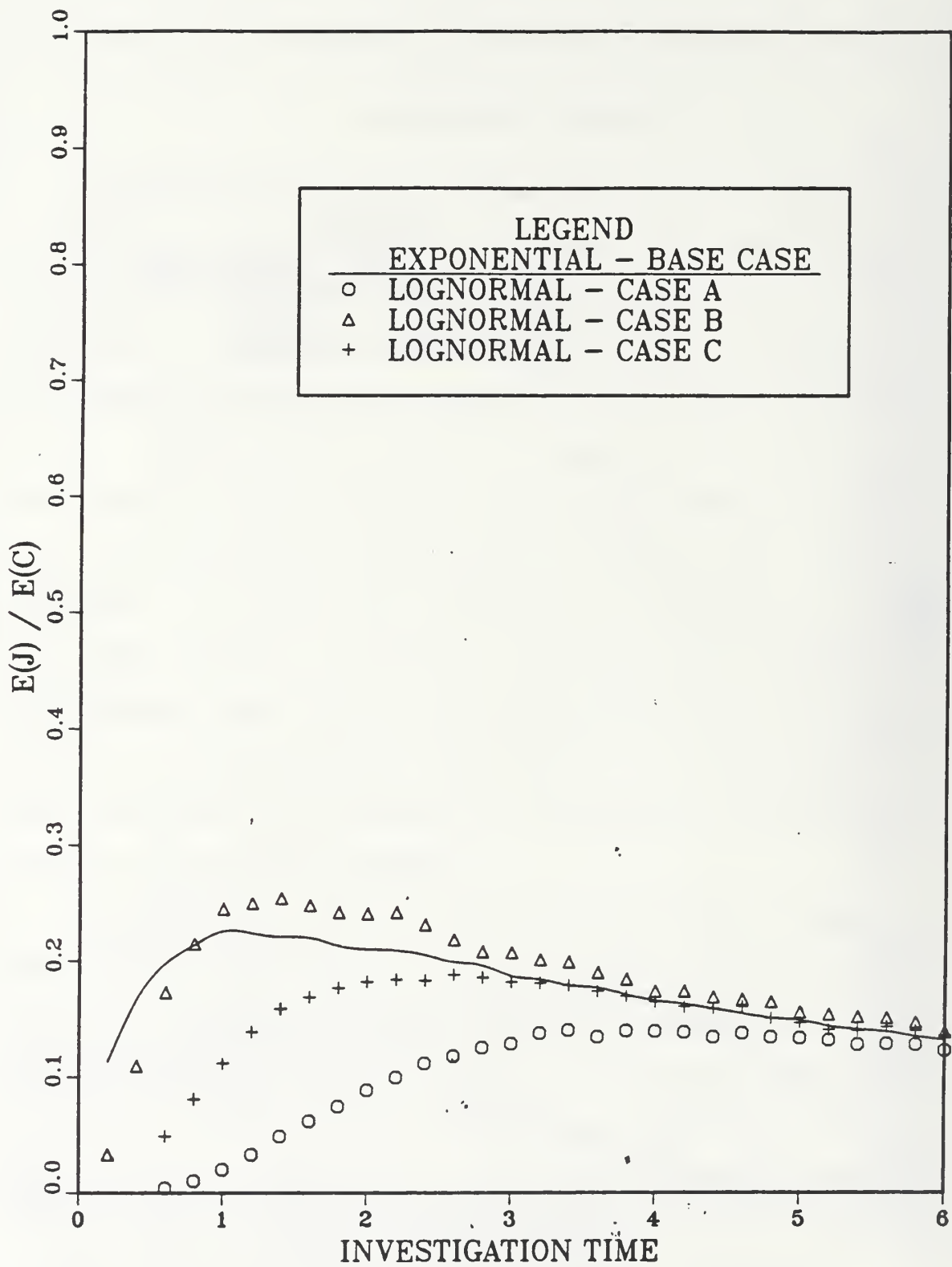


Figure 5: Reward Criteria 2 Sensitivity Analysis

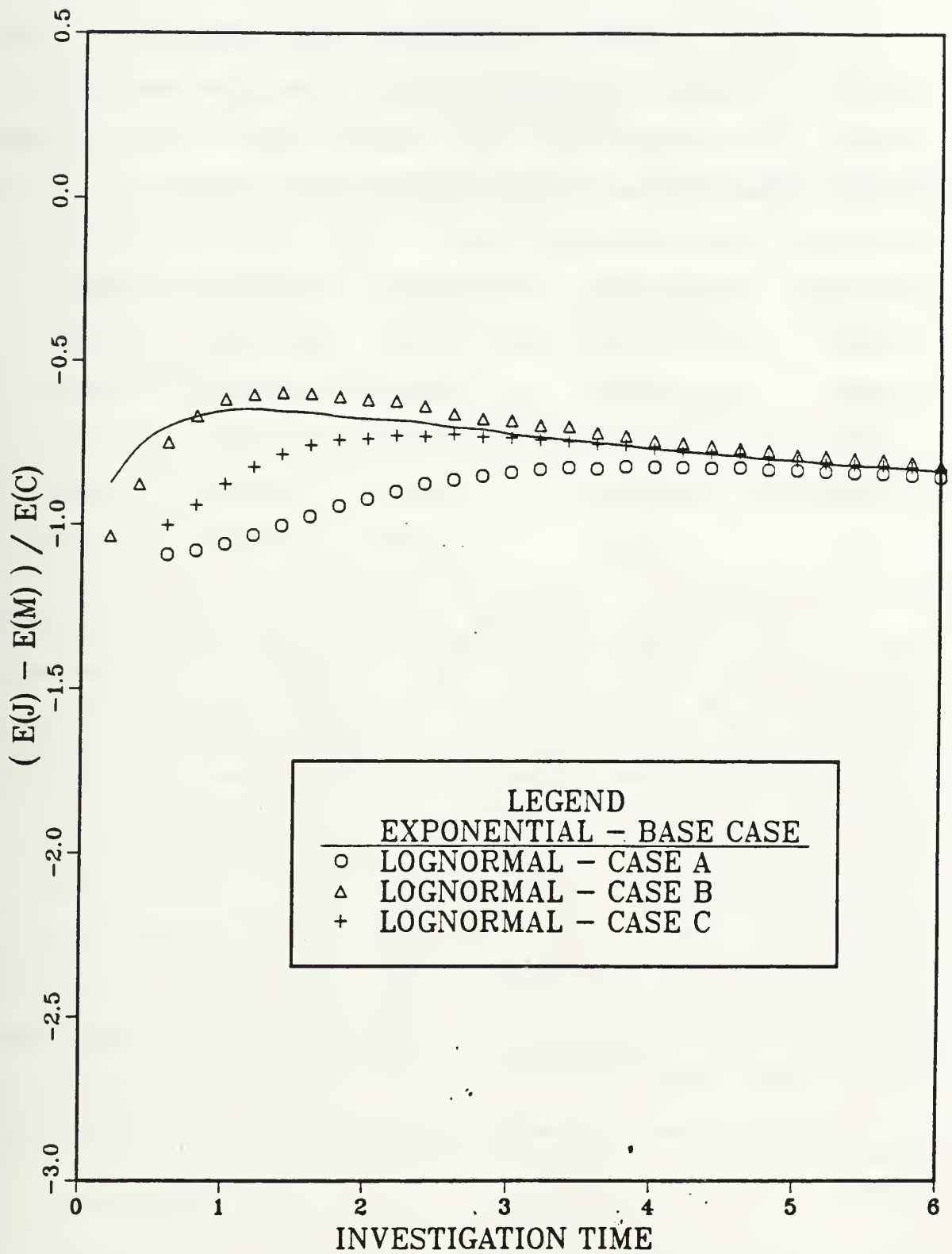


Figure 6: Reward Criteria 3 Sensitivity Analysis

In order to better understand this behavior, it is helpful to compare the quantiles of T, the time required to locate an incriminating unit given that J units are present, for the four distributions.

<u>Quantile</u>	<u>Base Case</u>	<u>Case A</u>	<u>Case B</u>	<u>Case C</u>
0.10	0.242	1.604	0.908	0.431
0.25	0.852	2.502	1.467	1.681
0.50	2.465	3.945	2.856	1.681
0.75	6.504	6.858	5.360	3.909
0.90	13.488	12.155	10.333	11.245

It can be seen that some distributions, in particular the lognormal, cases a, b, and c, have a far greater likelihood of producing large values of T relative to the others, such as the exponential distribution used in the examples. Since the time required to find the first unit of drugs greatly influences the cycle length, distributions generating larger values of T will produce significantly different results. To further verify the accuracy of the distributions produced by the simulation and the quantiles listed above, Appendix B contains the calculated quantiles for lognormal case A produced using analytical distribution theory.

## V. CONCLUSIONS

It has been demonstrated that a renewal-reward approach to modelling the Coast Guard drug interdiction process is feasible and that it is possible to determine an optimal investigation time. By considering several different reward criteria in the model, it is possible to quantify the costs with respect to the rest of the reward criteria when one criteria is selected and used to arrive at an optimum investigation time.

Of the two methods presented to obtain the optimum investigation time, the simulation approach is the most flexible. The example distributions and assumptions used in this thesis were selected because they allowed an analytical solution to be found. This allowed a comparison of results between the simulation and numerical solutions, thus verifying the simulation. As the model assumptions are changed and different probability distributions incorporated, the simulation can be easily modified to reflect these changes whereas an analytical solution may no longer be possible.

As the model is currently formulated, its usefulness is questionable due to the sensitivity to the underlying distribution of the time required to find drugs onboard a vessel given drugs are present. As demonstrated in the

sensitivity analysis even within a given family of distributions, the model is sensitive to the distribution parameters. In order for the model to produce realistic results, rigorous data analysis must be conducted to properly identify this distribution.

The basic service system model may have wider application, e.g., to combat modelling, where it may be desirable to investigate a potential target to estimate its value before committing limited resources. An adaption of the model may also be of help in allocating resources for mineral exploration.



## APPENDIX A

The simulation is written in Fortran 77. It consists of a main program and seven subroutines. Uniform [0,1] random numbers are provided by calling procedure GGUBFS located in the IBM IMSL single precision library. All real variables are computed using double precision to minimize rounding error. Output is directed to three separate units. Unit 2 contains the calculated rewards for each increment of investigation time using reward criteria 1, 2, and 3. Unit 3 contains the cycle length, drug quantity confiscated, and quantity of drugs missed for each increment of investigation time. Unit 4 contains detailed information regarding the cycle for each increment of investigation time. This information includes the total number of ships searched, the number of ships that are good, bad, bad but declared good, and bad identified as bad. Also included in unit 4 output is the time required to discover the drugs on a ship declared bad, the quantity of drugs onboard ships passing through the area while the CGV is searching a ship, the quantity of drugs missed due to short investigation time, and the quantity of drugs confiscated on a bad ship.

The main program controls the starting and final investigation time, investigation time increment width, and

the number of cycles per investigation time to be simulated. Cycle averages are computed and output directed to the three units discussed above.

Subroutine CYCLE simulates one patrol cycle and records all the relevant statistics during the cycle. Subroutine STIME generates the random search times required to find a ship from an exponential distribution. Subroutine DTIME generates the random times required to escort a ship back to base from an exponential distribution. Subroutine DRGQTY generates the random quantity of drugs on a BAD ship from a geometric distribution. Subroutine CLASS determines the classification of a vessel based on the deterministic value  $P_b$ . Subroutine OPCOST determines the quantity of drugs missed onboard other ships passing through the area while the CGV is investigating the current ship. Subroutine RTIME generates the random times required to find the first unit of drugs given that  $j$  units are present.

Four versions of this subroutine are listed; one for the exponential distribution and the other three for the lognormal case A, B, and C distributions.

```

C      THIS PROGRAM SIMULATES PATROL CYCLES FOR VARIOUS LENGTHS OF
C      INVESTIGATION TIMES AND CALCULATES THE VALUES FOR THE THREE
C      REWARD CRITERIA. FUNCTION GGUBFS FROM THE IMSLSP LIBRARY IS
C      USED TO GENERATE UNIFORM (0,1) NUMBERS.

      INTEGER I,K,MAXT,NCYCLE,Z
      REAL*8 L,C,J,M,INC,EC(500),EJ(500),EM(500),ITIME(500),TOEND,
& R1(500),R2(500),R3(500),TS,GS,BS,BSG,BSB,EIB,OP,MISS,EIG,
& CATCH,ES,ED,BJ,DSEED
      DSEED = 995317.D0
      MAXT = 4
      NCYCLE = 100
      INC = 0.20
      L = 0
      TOEND = MAXT/INC
      DO 200 K = 1,TOEND
        L = L + INC
        C = 0
        J = 0
        M = 0
        PRINT*, 'COMPUTING L = ',L
        DO 100 I = 1,NCYCLE
          CALL CYCLE(DSEED,L,C,J,M,TS,GS,BS,BSG,BSB,EIB,OP,MISS,
& CATCH,ES,ED,BJ,EIG)
100      CONTINUE
          Z=NCYCLE
          WRITE(4,*) L
          WRITE(4,523) TS/Z,GS/Z,BS/Z,BSG/Z,BSB/Z
          WRITE(4,524) EIB/Z,BJ/Z,OP/Z,MISS/Z,CATCH/Z
          WRITE(4,525) EIG/Z,ES/Z,ED/Z
          WRITE(4,*)
523      FORMAT(1X,5F9.4)
524      FORMAT(1X,5F9.4)
525      FORMAT(1X,3F9.4)

          TS =0
          BS =0
          BSG=0
          BSB=0
          GS =0
          EIB=0
          MISS=0
          CATCH=0
          ES =0
          ED =0
          BJ =0
          OP =0
          EIG=0

          ITIME(K) = L
          EC(K) = C/NCYCLE
          EJ(K) = J/NCYCLE
          EM(K) = M/NCYCLE
          R1(K) = 1/EC(K)
          R2(K) = EJ(K)/EC(K)
          R3(K) = (EJ(K)-EM(K))/EC(K)
          WRITE(2,19) ITIME(K),R1(K),R2(K),R3(K)
          WRITE(3,19) ITIME(K),EC(K),EJ(K),EM(K)
19      FORMAT (1X,F6.2,3F9.3)
200      CONTINUE
      STOP 'PROGRAM COMPLETE'
      END

      SUBROUTINE CYCLE(DX,LSTAR,CSTAR,JSTAR,MSTAR,TS,GS,BS,BSG,BSB,
& EIB,OP,MISS,CATCH,ES,ED,BJ,EIG).
      REAL*8 LSTAR,CSTAR,JSTAR,MSTAR,LAMDA1,LAMDA2,MU,BETA,S,R,QTY,
& D,LEAK,TIME,PBAD,TS,GS,BSG,BSB,EIB,OP,MISS,ES,ED,BJ,BS,CATCH,
& EIG,DX
      INTEGER NC
      CHARACTER*4 TYPE
      LAMDA1 = 1.10
      LAMDA2 = 0.3

```

```

      MU = 0.08
      BETA = 0.8
      PBAD = 0.2
20    TS=TS+1
      CALL STIME(DX,LAMDA1,S)
      ES=ES+S
      CALL CLASS(DX,PBAD,TYPE)
      IF( TYPE .EQ. 'GOOD') THEN
        EIG = EIG+LSTAR
        GS=GS+1
        QTY=0
        TIME = LSTAR
        CSTAR = CSTAR + TIME + S
        CALL OPCOST(DX,LAMDA1,TIME,BETA,PBAD,LEAK)
        MSTAR = MSTAR + LEAK
        OP=OP+LEAK
        GOTO 20
      ELSE IF( TYPE .EQ. 'BAD') THEN
        BS=BS+1
        CALL DRGQTY(DX,BETA,QTY)
        BJ=BJ+QTY
        CALL RTIME(DX,MU,QTY,R)
        IF(R .GT. LSTAR) THEN
          EIG=EIG+LSTAR
          BSG=BSG+1
          TIME = LSTAR
          CSTAR = CSTAR + TIME + S
          CALL OPCOST(DX,LAMDA1,TIME,BETA,PBAD,LEAK)
          MSTAR = MSTAR + LEAK + QTY
          OP=OP+LEAK
          MISS=MISS+QTY
          GOTO 20
        ELSE IF(R .LE. LSTAR) THEN
          EIB=EIB+R
          BSB=BSB+1
          CALL DTIME(DX,LAMDA2,D)
          ED=ED+D
          TIME = R + D
          CSTAR = CSTAR + TIME + S
          JSTAR = JSTAR + QTY
          CALL OPCOST(DX,LAMDA1,TIME,BETA,PBAD,LEAK)
          MSTAR = MSTAR + LEAK
          OP=OP+LEAK
          CATCH=CATCH+QTY
          GOTO 21
        ELSE
          STOP 'ERROR 1'
        ENDIF
      ELSE
        STOP 'ERROR 2'
      ENDIF
21    RETURN
      END

```

```

SUBROUTINE STIME(DRSEED,INPUT,OUTPUT)
REAL*8 INPUT,OUTPUT,RV,DRSEED
RV=GGUBFS(DRSEED)
OUTPUT = LOG(RV)/(-INPUT)
RETURN
END

```

```

SUBROUTINE DTIME(DRSEED,INPUT,OUTPUT)
REAL*8 INPUT,OUTPUT,RV,DRSEED
RV = GGUBFS(DRSEED)
OUTPUT = LOG(RV)/(-INPUT)
RETURN
END

```

```

SUBROUTINE DRGQTY(DRSEED,INPUT,OUTPUT)
REAL*8 DRSEED,INPUT,OUTPUT,RV
RV = GGUBFS(DRSEED)
OUTPUT = AINT(LOG(RV)/(LOG(INPUT)))+1
RETURN
END

```

```

SUBROUTINE CLASS(DRSEED,INPUT,OUTPUT)
REAL*8 DRSEED,INPUT,RV
CHARACTER*4 OUTPUT
RV = GGUBFS(DRSEED)
IF(RV .LE. INPUT) THEN
    OUTPUT = 'BAD'
ELSE
    OUTPUT = 'GOOD'
ENDIF
RETURN
END

```

```

SUBROUTINE OPCOST(DRSEED,INPUT1,INPUT2,P1,P2,OUTPUT)
REAL*8 DRSEED,INPUT1,INPUT2,P1,P2,OUTPUT,STOR,NSHIP
OUTPUT = 0
JO = 1/(1-P1)
NSHIP = INPUT2*INPUT1
OUTPUT = (P2*NSHIP)*JO
RETURN
END

```

```

C SUBROUTINE RTIME(DRSEED,INPUT1,INPUT2,OUTPUT)
EXPONENTIAL DISTRIBUTION - BASE MEAN & VARIANCE
REAL*8 DRSEED,INPUT1,INPUT2,OUTPUT,RV
RV = GGUBFS(DRSEED)
OUTPUT = LOG(RV)/(-INPUT1*INPUT2)
RETURN
END

```

```

C SUBROUTINE RTIME1(DRSEED,INPUT1,INPUT2,OUTPUT)
LOGNORMAL DISTRIBUTION - BASE MEAN & VARIANCE
REAL*8 INPUT1,INPUT2,OUTPUT,STDV,AVG,PI,LOW,U1,U2,GG,
& DRSEED
INTEGER II,FINISH
GG = LOG(2.0)
LOW = 999999
FINISH = INT(INPUT2)
STDEV = SQRT(GG)
AVG = -(LOG(INPUT1))- 0.5*GG
PI = 3.141592654
DO 662 II = 1,FINISH
    U1 = GGUBFS(DRSEED)
    U2 = GGUBFS(DRSEED)
    RV = SQRT(-2*LOG(U1))*COS(2*PI*U2)*STDEV+AVG
    RVT = EXP(RV)
    IF(RVT .LT. LOW) THEN
        LOW = RVT
    ENDIF
662 CONTINUE
OUTPUT = LOW
RETURN
END

```

```

C SUBROUTINE RTIME2(DRSEED,INPUT1,INPUT2,OUTPUT)
LOGNORMAL DISTRIBUTION - BASE MEAN & 2*VARIANCE
REAL*8 INPUT1,INPUT2,OUTPUT,STDV,AVG,PI,LOW,U1,U2,GG,
& DRSEED
INTEGER II,FINISH
GG = LOG(3.0)
LOW = 999999

```

```

        FINISH = INT(INPUT2)
        STDEV = SQRT(GG)
        AVG = -(LOG(INPUT1))- 0.5*GG
        PI = 3.141592654
        DO 662 II = 1,FINISH
            U1 = GGUBFS(DRSEED)
            U2 = GGUBFS(DRSEED)
            RV = SQRT(-2*LOG(U1))*COS(2*PI*U2)*STDEV+AVG
            RVT = EXP(RV)
            IF(RVT .LT. LOW) THEN
                LOW = RVT
            ENDIF
662    CONTINUE
        OUTPUT = LOW
        RETURN
        END

C      SUBROUTINE RTIME3(DRSEED,INPUT1,INPUT2,OUTPUT)
        LOGNROMAL DISTRIBUTION - BASE MEAN & 4*VARIANCE
        REAL*8 INPUT1,INPUT2,OUTPUT,STDV,AVG,PI,LOW,U1,U2,GG,
& DRSEED
        INTEGER II,FINISH
        GG = LOG(5.0)
        LOW = 999999
        FINISH = INT(INPUT2)
        STDEV = SQRT(GG)
        AVG = -(LOG(INPUT1))- 0.5*GG
        PI = 3.141592654
        DO 662 II = 1,FINISH
            U1 = GGUBFS(DRSEED)
            U2 = GGUBFS(DRSEED)
            RV = SQRT(-2*LOG(U1))*COS(2*PI*U2)*STDEV+AVG
            RVT = EXP(RV)
            IF(RVT .LT. LOW) THEN
                LOW = RVT
            ENDIF
662    CONTINUE
        OUTPUT = LOW
        RETURN
        END

```



```

C      THIS PROGRAM COMPUTES THE VALUES OF THE THREE REWARD CRITERIA
C      FOR VARIOUS VALUES OF L FOR COMPARISON WITH SIMULATION RESULTS
C
      REAL PB,LAMDA1,LAMDA2,BETA,MU,INC,ES,ED,FT,FTBAR,A,ABAR,
& L,TOEND,EC(100),EJ(100),EM(100),R1(100),R2(100),R3(100),
& ITIME(100),PG,B,BB,E,C1,C2,C3,C4,M1,M2,TEMP,J1,RATE
      INTEGER MAXT,K
      MAXT = 6
      INC = 0.20
      PB = .20
      PG = 1-PB
      LAMDA1 = 1.10
      LAMDA2 = 0.3
      ES=1/LAMDA1
      ED=1/LAMDA2
      BETA = 0.8
      MU = 0.08
      B=BETA
      BB=1-B
      RATE = (LAMDA1*PB)/BB
      TOEND = MAXT/INC
      L=0
      DO 100 K = 1,TOEND
        L=L+INC
        E=EXP(-MU*L)
        TEMP=1-(B*E)
        FTBAR = (BB*E)/TEMP
        FT=1-FTBAR
        ABAR=PB*FT
        A=1-ABAR
        C1= ES/ABAR
        C2=ED
        C3=(PB/ABAR)*((BB/(B*MU))*LOG(TEMP/BB)-(L*FTBAR))
        C4=(L*A)/ABAR
        J1=(BB/FT)*((1/(BB*BB))-(E/(TEMP*TEMP)))
        M1=(C3+ED)*RATE+((L*A*RATE)/ABAR)
        M2=BB*(E/(TEMP*TEMP))*(1/ABAR)*PB
        ITIME(K) = L
        EC(K) = C1+C2+C3+C4
        EJ(K) = J1
        EM(K) = M1+M2
        R1(K) = 1/EC(K)
        R2(K) = EJ(K)/EC(K)
        R3(K) = (EJ(K)-EM(K))/EC(K)
        WRITE(2,132) ITIME(K),R1(K),R2(K),R3(K)
        WRITE(3,132) ITIME(K),EC(K),EJ(K),EM(K)
        WRITE(4,*) L
        WRITE(4,133) C1,C2,C3,C4
        WRITE(4,134) J1
        WRITE(4,135) M1,M2
132      FORMAT (1X,F6.2,3F9.3)
133      FORMAT (1X,4F12.5)
134      FORMAT (1X,F12.5)
135      FORMAT (1X,2F12.5)
        WRITE(4,*)
100    CONTINUE
      STOP 'PROGRAM EXECUTION COMPLETE'
      END

```



## APPENDIX B

Recall that the probability distribution for the quantity of drugs on a vessel given that it is bad is:

$$b_j = (1 - \beta)\beta^{j-1}; \quad \beta = 0.8, \quad j = 1, 2, 3, \dots$$

Let

$$P\{T > t\} \equiv \overline{F_T}(t) = \sum_{j=1}^{\infty} (1 - F_Z(t))^j b_j$$

so

$$\overline{F_T}(t) = \frac{(1 - \beta)\overline{F_Z}(t)}{1 - \beta\overline{F_Z}(t)}$$

and

$$P\{T \leq t\} = 1 - \frac{(1 - \beta)\overline{F_Z}(t)}{1 - \beta\overline{F_Z}(t)} = \frac{1 - \overline{F_Z}(t)}{1 - \beta\overline{F_Z}(t)}$$

now let

$$\frac{1 - \overline{F_Z}(t_p)}{1 - \beta\overline{F_Z}(t_p)} = p$$

which can be written as

$$\overline{F_Z}(t_p) = \frac{1 - p}{1 - \beta p}$$

and

$$F_Z(t_p) = 1 - \overline{F_Z}(t_p) = \frac{p(1 - \beta)}{1 - \beta p}$$

Assuming  $F_z(t)$  is lognormal with mean = 2.1792 and standard deviation = 0.8326 as in distribution Case A

$$\Phi \left( \frac{\ln t_p - \mu}{\sigma} \right) = \frac{p(1-\beta)}{1-\beta p}$$

therefore

$$t_p = \text{EXP} \left[ \mu + \sigma Z_{\frac{p(1-\beta)}{1-\beta p}} \right]$$

where  $Z_{\frac{p(1-\beta)}{1-\beta p}}$  is taken from tables of the Standard Normal distribution.

Using the equation for  $t_p$ , the following table of quantiles was generated:

<u>p</u>	<u><math>t_p</math></u>
0.10	1.65
0.25	2.46
0.50	3.95
0.75	6.78
0.90	11.99

It can be seen that these quantile values agree closely with those presented in the sensitivity analysis.

## REFERENCE

Ross, S. M., Introduction To Probability Models, Academic Press, Inc., 1985.

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